Physics Lab Report: Data Analysis

Abstract

In this report, value of acceleration due to gravity was obtained with various methods giving importance to the approach and the error analysis of these methods. From the simple equation of time period of a string, by keeping a specific length and by measuring time periods, the value of acceleration due to gravity by found out by simple substitution and by graphing methods. Different methods of errors, analysis, propagation and steps to avoid them is also discussed.

Introduction

Time period of a pendulum:

The equation for time period of a simple harmonic motion is derived from the equations of simple harmonic motion. For a pendulum, the restoring force is the force perpendicular to the string at any position.

$$F = mg \times sin(\theta) \quad (1)$$

Comparing this with the equation of a simple harmonic oscillator force equation, the value of *k* can be found. Note that θwill tend to be equal to sin(θ) as θnears zero.

$$k(l\theta) = mgsin(\theta);\ \theta \simeq sin(\theta) \quad (2),(3)$$

$$k = mg/l \quad (4)$$

Comparing this with the time period equation for a simple harmonic oscillator, we obtain the equation of time period of a simple pendulum

$$T = 2\pi\sqrt{m/k} \quad (5)$$

$$T = 2\pi\sqrt{l/g} \quad (6)$$

Assumptions:

1. The amplitude should be small for the quadratic potential well
2. As external forces has not been considered, contribution of air drag is ignored
3. The string is inextensible and light

Error Analysis:

For the experiment, there are plenty of sources of errors which is accounted for in the results. There are two variables which have to be measured, both of which produces its own errors. The length of the string measured by the ruler will have an error which will have errors contributed by both the ends.

$$\Delta l = \sqrt{(\Delta l_1)^2 + (\Delta l_2)^2} \quad (7)$$

While measuring time, similarly there will be an error between the start point and the end point

$$\Delta T = \sqrt{(\Delta T_1)^2 + (\Delta T_2)^2} \quad (8)$$

But this error can be practically be reduced by increasing the number of oscillations on which time is measured. For example, if a time of $T_{Total} + \Delta T$is measured for *n* oscillations, time period for 1 oscillation will be $T_{Total}/n + \Delta T/n$; implying that as the number of oscillations increase, the uncertainties decrease.

However it should be noted that, for each oscillation the time period keeps on decreasing by a small factor due to dissipative forces i.e friction and drag. So the number of oscillations, *n* should be limited to a reasonable number to limit the total reduction due to dissipation.

The error of g for task 1 and task2 can be calculated through the formula

$$\Delta g = (2\pi\sqrt{L}/T) \times \sqrt{(\Delta L/L)^2 + (\Delta T/T)^2} \qquad (9)$$

For **task 3**, for finding the best fit by the method of least squares, the slope can be found out by the equation 10. This is not the standard equation as we require the y intercept to be zero.

$$m = \Sigma x_i y_i / \Sigma {x_i}^2 \qquad (10)$$

the value of *g* can be obtained by the equation 10 and the error using the equation 11

$$g = 1/m^2 \qquad (11)$$

$$\Delta g = 2\Delta m/m^3 \qquad (12)$$

Practical uses:

The pendulum is a simple device with easy to analyse properties and hence has a lot of applications

1. Measuring time: The traditional and direct application of a pendulum is to keep track of time due to it's nearly constant time period.
2. Measuring gravity: Local acceleration due to gravity changes from place to place and a smooth well constructed pendulum can be used to measure and compare the acceleration due to gravity similar to the experiment
3. Measuring acceleration: Pendulum clocks are were not widely used in ships because of its varying time period with acceleration. This property can be used to measure acceleration of a ship as the acceleration that contributes to the time period will be a superposition of both forward/backward acceleration and acceleration due to gravity. By finding out the magnitude of the effective acceleration, the forward/backward acceleration can be calculated.

Method

Task 1:

Set the length of the pendulum to be 15 cm. Measure time for one oscillation. Repeat time measurement 5 times. Find out the value of *g* by substituting the average value of time period. Find the error for this value

Task 2:

Set the length of the pendulum to be 15 cm. Measure time for 10 oscillations. From this, find the value of time period for one oscillation and its error. From this, obtain the value of *g* along with its error

Task 3:

Perform experiments with lengths of the pendulum equal to 7 cm, 19 cm, 21 cm, 23 cm, and 25cm. Find the time period for each by recording time for 10 oscillations and dividing by ten. After obtaining this data, plot $y = T$ and $x = 2\pi\sqrt{L}$. Obtain the value of g by the following methods

1. Line of best fit method 1: Draw 2 lines with highest and lowest gradients and draw a line with the average of the two gradients. The slope will be equal to $1/\sqrt{m}$
2. Method of least squares: Substitute the values to find the slope with the least square error.

In both of these cases obtain the uncertainties

Precautions:

1. Make sure the length measured is between the start point and the centre of mass of the weight
2. Keep the amplitude small to satisfy the assumption for equation 4
3. Make sure to reduce external forces like mechanical friction, air drag and any other external damping

Results

For **task 1**, 5 measurements are taken and is given in Table 1. The uncertainty is calculating by dividing the range of the measurements by 2 since the data set is small. The uncertainty of the average is calculated by dividing this uncertainty of the set by the square root of the number of observations i.e. 5.

Table1: Task 1

L = 15 cm	1ST	2ND	3ND	4TH	5TH
	0.68	0.71	0.73	0.69	0.7
Average Periods	0.694				
The uncertainty of the average	The uncertainty of the set: 0.025 The uncertainty of the average : 0.0112				
Reported value	0.694+-0.0112				
g	12.295+-0.229				

For **task 2,** 10 measurements are made as shown in Table 2. The uncertainty of the set is standard deviation in this case since sufficiently large data set is used. The uncertainty of the average is obtained by dividing the uncertainty of the set by square root of the number of measurements i.e. 10

Table 2: Task 2

L = 15 cm	1ST	2ND	3ND	4TH	5TH	6TH	7TH	8TH	9TH	10th
	0.875	0.869	0.872	0.878	0.884	0.875	0.862	0.866	0.872	0.878
Average Periods	0.873									
The uncertainty of the average	The uncertainty of the set: 0.006 The uncertainty of the average : 0.002									
Reported value	0.873+-0.002									

g	7.77+-0.075

For **task 3**, the value of *g* are found out in 2 ways - line of best fit method and the least square method. The measurements are given in Table 3.

Table 3:Task 3 measurements

	L=15 cm	L = 17 cm	L = 19 cm	L = 21 cm	L = 23 cm	L = 25 cm
1	0.875	0.918	0.953	1	1.021	1.06
2	0.869	0.909	0.959	1.01	1.022	1.06
3	0.872	0.919	0.953	1.003	1.025	1.06
4	0.878	0.912	0.956	1.003	1.022	1.062
5	0.884	0.918	0.956	1.005	1.028	1.069
6	0.875	0.916	0.956	1.006	1.025	1.069
7	0.862	0.917	0.956	1	1.028	1.062
8	0.866	0.92	0.956	1.004	1.025	1.06
9	0.872	0.916	0.956	1.007	1.023	1.06
10	0.878	0.92	0.956	1.001	1.025	1.066
Average	0.8731	0.9165	0.9557	1.0039	1.0244	1.0628
Range	0.022	0.011	0.006	0.01	0.007	0.009
Uncertainty of the set	0.006	0.004	0.002	0.003	0.002	0.004
Uncertainty of avg	0.0019	0.0013	0.0006	0.0009	0.0006	0.0013

Line of Best Fit Method 1:

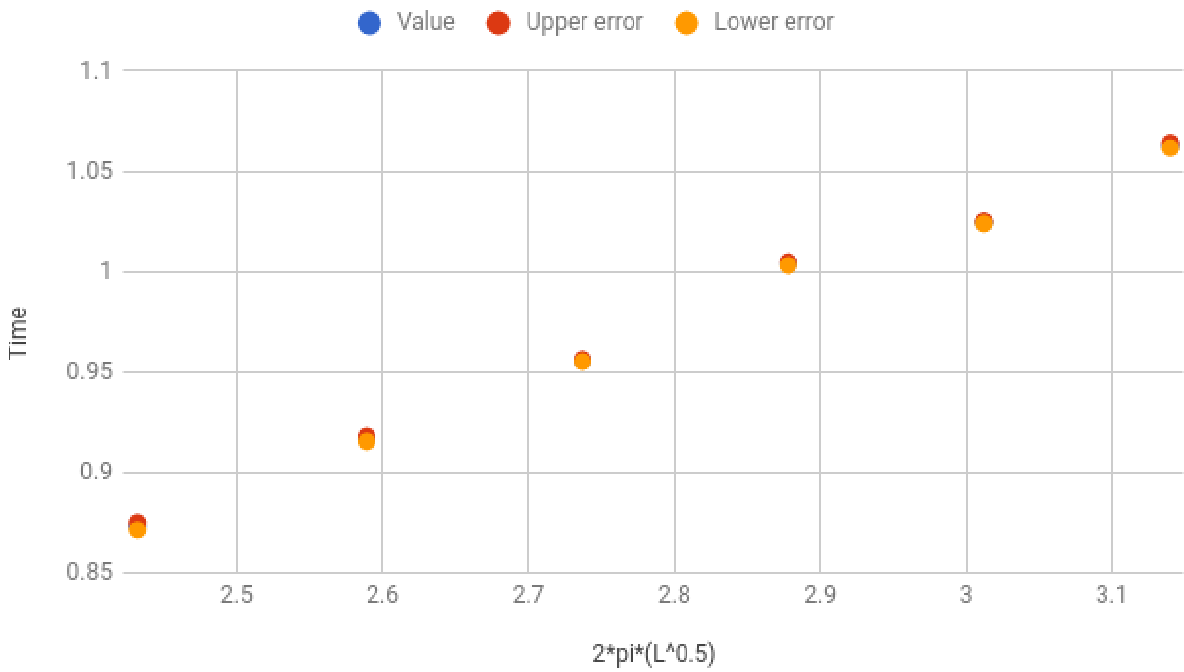

Fig 1. Plot between time and $2\pi\sqrt{L}$

Highest gradient = 0.352 s/(m^0.5)
Lowest gradient = 0.252 s/(m^0.5)
Average gradient = 0.302 s/(m^0.5)
Error in gradient = 0.05 s/(m^0.5)

Therefore value of g obtained by this method (using equation 11) = 10.964 m/(s^2)
Using equation 12 $\Delta g \; = 2 * 0.05/0.302^3$= 3.630 m/(s^2)

Least square method:

Table 4: Calculating slope through least square method

	x(2*pi*sqrt(L))	y(Time)	xy	x^2	mx	S_d^2
	2.432	0.8731	2.1234	5.9146	0.844	0.00085
	2.589	0.9165	2.3728	6.7029	0.899	0.00031
	2.737	0.9557	2.6158	7.4912	0.95	0.00003
	2.878	1.0039	2.8892	8.2829	0.999	0.00002
	3.012	1.0244	3.0855	9.0721	1.045	0.00042
	3.14	1.0628	3.3372	9.8596	1.09	0.00074

Sum	16.788		16.4239	47.3233		0.00237

m = 0.347 s/(m^0.5)

Error of *m* at 68% confidence level = 0.041 s/(m^0.5)
Error of *m* at 95% confidence level = 0.081 s/(m^0.5)

Value of *g* = 8.305m/(s^2)
Error of *g* at 68% confidence level = 1.962 m/(s^2)
Error of *g* at 95% confidence level = 3.877 m/(s^2)

The actual value of *g* =9.81 m/(s^2)

For task 1 and task 2, the values obtained lies well out of the range of errors. For task 3, best line method 1, the error between the obtained value and the actual value is minimum but the uncertainty is the highest. For task 3, least square method gives value of *g* to be within the uncertainty yet the error is high.

Analysis

The large errors are caused by the least count of errors of the ruler, the human errors in measuring time and other system errors. The contributors to the large error might be because of testing with large amplitudes or mechanical friction. Further, there can be a random error because of parallax while measuring length keeping in mind the accurate starting position and the centre of mass of weight. This can be reduced by increasing the number of repetitions.

Task 1 is highly inaccurate for its method of measuring time periods. The reaction time produces human error which can produce large errors. For this method, a higher accurate value can be produced by repeating the experiment more times.

Task 3 provided values of acceleration due to gravity within its errors. However error for the methods was very high. The uncertainties and accuracies can be improved by taking time periods of more length values. Higher the repetitions, higher are the time and the relation averaged.

Conclusions

The four values of *g* obtained are spread around the established value of *g*=9.81 m/(s^2). From the third task, both the values are within the uncertainty limits. This proves that the hypothesized equation should be true to an approximate value.

For further explorations, an experiment could be to check the pendulum on damping conditions. Others include testing the time period on larger amplitudes and with non-uniform weights to establish center of mass.

The experiment even though did not predict accurate values of *g*, it gave values within errors and provided an introduction to the statistical error analysis. Various types of errors, methods of error analysis and propagation has been explored and experimented with. Methods to reduce these errors also have been discussed.