Physics Assignment Help

1.

a)

We have total mass of white dwarf as M. And all this mas come from N/2 nuclei's and every nuclei have mass $m_n = 4m_p \approx 4\ amu$.

Then:

$$M = \frac{N}{2}m_n = \frac{N}{2}4m_p = 2Nm_p, \ \ N = \frac{M}{2m_p} \approx \frac{10^{30}}{2 * 1.66 * 10^{-27}} \sim 3.02 * 10^{56}$$

Energy of Fermi for electrons in system is:

$$e_F = \frac{\hbar^2}{2m_e}\left(\frac{3\pi^2 N}{V}\right)^{\frac{2}{3}}, \quad N = \frac{M}{2m_p}, \quad V = \frac{M}{\rho} \quad then\ e_F = \frac{\hbar^2}{2m_e}\left(\frac{3\pi^2\rho}{2m_p}\right)^{\frac{2}{3}} \approx 4.79 * 10^{-12}$$

b)

Total energy can be calculated as for $E(p) = \sqrt{p^2c^2 + m^2c^4}$,

$N(E) = \frac{V}{6\pi^2}\left(\frac{2mE}{\hbar^2}\right)^{\frac{3}{2}}$:

$$U = \int_0^{p_F} dp \frac{dN}{dp} E(p) = V\int_0^{p_F} \frac{4\pi p^2\sqrt{p^2c^2 + m^2c^4}dp}{2m(2\pi\hbar)^3},$$

Then, is $x_F = \frac{p_F}{m_e c}$, $\quad dp = m_e c dx$

$$U = V\int_0^{p_F} \frac{4\pi p^4 dp}{2m(2\pi\hbar)^3} = V\frac{4\pi}{2m(2\pi\hbar)^3}\int_0^{x_F}(m^5c^5)x^2\sqrt{x^2+1}\,dx = \frac{Vm^4c^5}{\pi^2\hbar^3}\int_0^{x_F} x^2\sqrt{x^2+1}\,dx$$

Fermi momentum is :

$$p_F = \sqrt{2m_e E_F} = \sqrt{2m_e\frac{\hbar^2}{2m_e}\left(\frac{3\pi^2 N}{V}\right)^{\frac{2}{3}}}$$

Then, knowing that $V = \frac{4}{3}\pi R^3$, $\ N = \frac{M}{2m_p}$

$$p_F = \hbar\sqrt{\left(\frac{3\pi^2 N}{V}\right)^{\frac{2}{3}}} = \hbar\sqrt{\left(\frac{9\pi^2 M}{8m_p\pi R^3}\right)^{\frac{2}{3}}} = \frac{\hbar}{2R}\left(\frac{9\pi M}{m_p}\right)^{\frac{1}{3}}$$

c)

The pressure is a derivative:

$$P = -\left(\frac{\partial U}{\partial V}\right)_N$$

Then:

$$P = -\frac{\partial\left(\frac{Vm^4c^5}{\pi^2\hbar^3}\int_0^{x_F} x^2\sqrt{x^2+1}\,dx\right)}{\partial V} = -\frac{\partial\big(Vf(V)\big)}{\partial V} = -f(V) - V\frac{\partial\big(f(V)\big)}{\partial V}$$
$$= -\frac{m^4c^5}{\pi^2\hbar^3}\int_0^{x_F} x^2\sqrt{x^2+1}\,dx - I_2$$

Lets take a look at this second part I_2:

$$I_2 = \frac{m^4c^5}{\pi^2\hbar^3}V\frac{\partial}{\partial V}\left(\int_0^{x_F(V)} x^2\sqrt{x^2+1}\,dx\right) = \frac{m^4c^5}{\pi^2\hbar^3}V\frac{\partial}{\partial V}\left(F\big(x_F(V)\big) - F(0)\right)$$

And for

$$x_F = \frac{\hbar}{m_e c}\left(\frac{3\pi^2 N}{V}\right)^{\frac{1}{3}}, \quad F\big(x_F(V)\big) = \left(\frac{\hbar}{m_e c}\right)^2\left(\frac{3\pi^2 N}{V}\right)^{\frac{2}{3}}\sqrt{\left(\frac{\hbar}{m_e c}\right)^2\left(\frac{3\pi^2 N}{V}\right)^{\frac{2}{3}} + 1}$$

Then we can get :

$$I_2 = -\frac{m^4c^5}{\pi^2\hbar^3}\left(\frac{1}{3}x_F^3\sqrt{x_F^2+1}\right)$$

And finally, the pressure:

$$P = \frac{m^4c^5}{\pi^2\hbar^3}\left(\frac{1}{3}x_F^3\sqrt{x_F^2+1} - \int_0^{x_F} x^2\sqrt{x^2+1}\,dx\right)$$

d)

Lets see:

$$\frac{GM^2}{R}, \; G \sim \frac{m^3}{kg * s^2}, \quad Then \;\; \frac{GM^2}{R} \sim \frac{m^3\,kg^2}{kg * s^2\;\; m} \sim \frac{m^2 * kg}{s^2}$$

And this is dimention of energy

e)

f)

Here we have:

$$-\alpha \frac{GM^2}{R} = \int_{\infty}^{R} 4\pi P(r) r^2 dr$$

Then:

$$\frac{d\left(-\alpha \frac{GM^2}{R}\right)}{dR} = \alpha \frac{GM^2}{R^2}$$

While:

$$\frac{d\left(\int_{\infty}^{R} 4\pi P(r) r^2 dr\right)}{dR} = F(R) - F(\infty) = 4\pi P(R) R^2 - 0$$

Then, we can write:

$$4\pi P(R) R^2 = \alpha \frac{GM^2}{R^2}, \quad P = \alpha \frac{GM^2}{4\pi R^4}$$

g)

Now we have:

$$\alpha \frac{GM^2}{4\pi R^4} = \frac{m^4 c^5}{\pi^2 \hbar^3}\left(\frac{1}{3} x_F^3 \sqrt{x_F^2 + 1} - \int_0^{x_F} x^2 \sqrt{x^2 + 1}\, dx\right)$$

Using:

$$\frac{1}{3} x_F^3 \sqrt{x_F^2 + 1} - \int_0^{x_F} x^2 \sqrt{x^2 + 1}\, dx = \frac{x_F^5}{15}, \quad for\ x_F \ll 1$$

$$\alpha \frac{GM^2}{4\pi R^4} = \frac{m^4 c^5}{\pi^2 \hbar^3} \frac{x_F^5}{15}, \quad x_F = \frac{p_F}{m_e c}, \quad \alpha \frac{GM^2}{4\pi R^4} = \frac{p_F^5}{15\pi^2 \hbar^3 m_e}, \quad p_F = \frac{\hbar}{2R}\left(\frac{9\pi M}{m_p}\right)^{\frac{1}{3}}$$

Then:

$$\alpha\frac{GM^2}{4\pi R^4}=\left(\frac{\hbar}{2R}\right)^5\left(\frac{9\pi M}{m_p}\right)^{\frac{5}{3}}\frac{1}{15\pi^2\hbar^3 m_e},\quad \alpha GM^2=\frac{\hbar^2}{8R}\left(\frac{9\pi M}{m_p}\right)^{\frac{5}{3}}\frac{1}{15\pi m_e}$$

Then:

$$R=\frac{\hbar^2 3}{40m_p m_e}\left(\frac{9\pi M}{m_p}\right)^{\frac{2}{3}}\frac{1}{\alpha GM}=\frac{3\hbar^2}{40m_p m_e\alpha G}\left(\frac{9\pi}{m_p}\right)^{\frac{2}{3}}M^{-\frac{1}{3}}$$