1. Evaluate the following:

$\int xsin(3x)dx$

$\int udv = uv - \int vdu, where\ u = x\ and\ dv = \sin(3x)$

$x\left[-\frac{1}{3}\cos(3x)\right] - \int -1/3\cos(3x)\,dx$

$\frac{-xcos(3x)}{3} - \int \frac{-\cos(3x)\,dx}{3}$

$Since - 1\ is\ constant\ with\ respect\ to\ x, the\ integral\ of\ \frac{-\cos(3x)}{3}\ with\ respct\ to\ x\ is$

$$-\int \frac{\cos(3x)\,dx}{3}$$

$\frac{-\cos(3x) -}{3} - \int \frac{\cos(3x)\,dx}{3}$

$\frac{-xcos(3x)}{3} + \int \frac{\cos(3x)\,\mathrm{dx}}{3}$

$Since\ \frac{1}{3}\ is\ constant\ with\ respect\ to\ x\ the\ integral\ of\ \frac{\cos(3x)}{3}\ with\ respect\ to\ x\ is\ \frac{1}{3}\int \cos(3x)\,dx$

$\frac{-xco(3x)}{3} + \frac{1}{3}\int \cos(3x)\,dx$

$let\ u = 3x. The\ du = 3dx, so\ \frac{1}{3}\ du = dx$

$Rewrite\ the\ using\ u\ and\ du\ \frac{-xcos(3x)}{3} + \frac{1}{3}\int \cos(u)\frac{1}{3}du$

$\frac{-xcos(3x)}{3} + \frac{1}{3}\int \frac{\cos(u)\,du}{3}$

$Sinnce\ \frac{1}{3}\ is\ constant\ with\ respect\ to\ u, the\ integral\ of\ \frac{\cos(u)}{3}\ with\ respect\ to\ u\ is\ \frac{1}{3}\int \cos(u)\,du$

$\frac{-xcos(3x)}{3} + \frac{1}{3}\left(\frac{1}{3}\int \cos(u)\,du\right)$

Combine fractions

$\frac{-xcos(3x)}{3} + \frac{1}{9}\int \cos(u)\,du$

$The\ integral\ of\ \cos(u)\ with\ respect\ to\ u\ is\ \sin(u)\frac{-xcos(3x)}{3} + \frac{1}{9}(\sin(u) + c)$

Simplify the answer.

$$-\frac{1}{3}xcos(3x)+\frac{1}{9}\sin(3x)+c$$

2. A=I

$$=\begin{bmatrix}\cos a - & \sin a \\ \sin a & \cos a\end{bmatrix}=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

Comparing the elements

$$\cos a = 1$$
$$cosa = \cos(2n\pi)$$
$$\left|\begin{matrix}-\sin a = 0 \\ \sin a = 0 \\ a = n\pi\end{matrix}\right| \left|\begin{matrix}sina = 0 \\ a = n\pi\end{matrix}\right| \begin{matrix}-\cos a = 1 \\ \cos a = -1\end{matrix}$$

$$a = (2n+1)\pi \ldots\ldots(2)$$

$where\ n\epsilon z$

From (1)

A is even

But from (2)

A is odd

There is no such value of which both 1 and 2 can be true.

: · A≠1

3. Given curve is

$y^2 = 4x$

Differentiating $w.r.tx$

$$\frac{d(y^2)}{dx}=\frac{d(4x)}{dx}$$

$$\frac{d(y^2)}{dy}x\frac{dy}{dx}=4$$

$$2y \times \frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{4}{2y}$$

$$\frac{dy}{dx} = \frac{2}{y}$$

$$Given\ line\ is\ y = x + 1\ The\ above\ line\ is\ of\ the\ form\ \ y = mx + c\ when\ m\ is\ slope\ of\ line\ y = x + 1\ is\ 1$$

Now slope of tangent = slope of line

$$\frac{dy}{dx} = 1$$

$$\frac{2}{y} = 1$$

2 = y

Finding x when y = 2

$$y^2 = 4x$$

$$(2)^2 = 4x$$

X=1

Hence the points $(x, y) = (1,2)$

4. Step 1 : find $\frac{dy}{dx}$

$$(x^2 - y^2)dx + 2xydy = 0$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \dots\dots\dots\dots\dots..(i)$$

Step 2: putting $f(x, y) = \frac{dy}{dx}$ $and\ finding\ f(\lambda x, \lambda y)$

$$\mathrm{f(x, y)} = \frac{\mathrm{y^2 - x^2}}{\mathrm{2xy}}$$

$$f(\lambda x, \lambda y) = \frac{(\lambda y)^2 - (\lambda x)^2}{2\lambda x * \lambda y} = \frac{x^2y^2 - \lambda^2 x^2}{\lambda^2 * 2xy} = \frac{\lambda^2(y^2 - x^2)}{\lambda^2 * 2xy} = \frac{y^2 - x^2}{2xy} = f(x, y)$$

$$f(\lambda x, \lambda y) = f(x, y) = \lambda^0 f(x, y).$$

Hence, $f(x, y) is\ a\ homogenous\ fuction\ of\ with\ degree\ zero$
$\frac{dy}{dx}\ is\ a\ homogenous\ differential\ equation.$

Step 3

$$Solving\ \frac{dy}{dx}\ by\ putting\ y = vx, put\ y = vx$$

$$Differentiating\ w.r.tx$$

$$\frac{dy}{dx} = x\frac{dv}{dx} + \frac{vdx}{dx}$$

$$\frac{dy}{dx} = x\frac{dv}{dx} + v$$

$$putting\ value\ of\ \frac{dy}{dx}\ and\ y = vx\ in\ (i)$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

$$\frac{xdv}{dx} + v = \frac{(xv)^2 - x^2}{2x(xv)}$$

$$\frac{xdv}{dx} + v = \frac{x^2v^2 - x^2}{2x^2v}$$

$$\frac{xdv}{dx} = \frac{-x^2v^2 - x^2}{2x^2v}$$

$$\frac{dv}{dx} = -\frac{1}{3}\left(\frac{x^2(v^2 + 1)}{2x^2v}\right)$$

$$\frac{2vdv}{v^2 + 1} = \frac{-dx}{x}$$

Integrating both sides

$$\int \frac{2v}{v^2 + 1} dv = \int \frac{-dx}{x}$$

$$\int \frac{2v}{v^2+1}dv = -log|x| + c \dots\dots\dots\dots..(ii)$$

$$putting\ \ t = v^2 + 1$$

$$diff.w.r.tv$$

$$\frac{d}{dv}(v^2+1) = \frac{dt}{dv}$$

$$2v = \frac{dt}{dv}$$

$$dv = \frac{dt}{2v}$$

Now from (ii)

$$\int \frac{2v}{t}\frac{dt}{2v} = -log|x| + c\ from\ (ii)$$

$$\int \frac{dt}{t} = -log|x| + c$$

$$\log|t| = -log|x| + c$$

$$putting\ t = v^2 + 1$$

$$log|v^2+1| = -log|x| + c$$

$$log|v^2+1| + log|x| = c$$

$$log|x(v^2+1)| = c$$

$$putting\ v = \frac{y}{x}$$

$$log\left|[({}^{y}/_{x})^2 + 1]x\right| = c$$

$$putting\ c = \log c$$

$$log\left|\frac{y^2+x^2}{x}\right| = \log c$$

$$y^2 + x^2 = cx$$

Is the general solution of the given differential equation.

5. $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{pmatrix}$

$|A| = 4$

$$Ax = \begin{pmatrix} 7 & -1 & 2 \\ -5 & 4 & -5 \\ 12 & -1 & 3 \end{pmatrix}$$

$|Ax| = 8$

$$Ay = \begin{pmatrix} 1 & 7 & 2 \\ 3 & -5 & -5 \\ 2 & 12 & 3 \end{pmatrix}$$

$|Ay| = 4$

$$Az = \begin{pmatrix} 1 & -1 & 7 \\ 3 & 4 & -5 \\ 2 & -1 & 12 \end{pmatrix}$$

$|Az| = 12$

$$x = \frac{|Ax|}{|A|} = \frac{8}{4} = 2$$

$$y = \frac{|Ay|}{|A|} = \frac{4}{4} = 1$$

$$z = \frac{|Az|}{|A|} = \frac{12}{4} = 3$$